MA-101/1841

B. Tech. (Semester-I) Examination- 2012

Math-I

Time: Three Hours

Maximum Marks: **100**

Note: Attempt questions from all the sections.

Section –A

Note: Attempt any ten questions. Each question carries four marks. **(4x10=40)**

1. Trace the curve $r^2 = a^2\ Cos2\theta$.

2. If $u = Sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$

   Show that $x\ \frac{\partial u}{\partial x} + y\ \frac{\partial u}{\partial y} + z\ \frac{\partial u}{\partial z} = -3 \tan u$

3. If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, *show that*

   $x^2\ \frac{\partial u}{\partial x} + y^2\ \frac{\partial u}{\partial y} + z^2\ \frac{\partial u}{\partial z} = 0$

4. If u, v are functions of r, s where r, s are functions of x, y then prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$

5. Compute an approximate value of $(1.04)^{3.01}$

6. Find the extreme values of function $x^3 + y^3 - 3axy$ where $a < 0$.

7. Verify Cayley-Hamilton theorem for matrix

$$A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

8. Find the inverse of matrix by Employing Elementary transformation

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

AI = A

9. Reduce matrix to normal form and find rank.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$$

10. Evaluate $\iint_R xy\,dxdy$ over the region bounded by $x = 0,\ y = 0,\ x + y = a$

1. Evaluate by changing into polar coordinates

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$$

12. Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

13. State Gauss divergence theorem.

14. What is greatest rate of increase of $u = xyz^2$ at the point (1,0,3)?

15. For any closed surface S, prove that $\iint_s curl\,\vec{F}\,.\hat{n}\,ds = 0$

Section –B

Note: Attempt any three questions. Each question carries twenty marks. **(20x3=60)**

1. (a) If $y = e^{m\cos^{-1}x}$, show that
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$
and calculate $y_n(0)$.

(b) Expand $\tan^{-1}\frac{y}{x}$ about (1,1). Hence compute $f(1.1,0.9)$ approximately.

2. (a) If u, v, w are the roots of cubic
$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0\ in\ \lambda$
then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

(b) The temperature T at any point (x,y,z) in space is T= 400 xyz^2. Find the highest temperature at surface of unit sphere $x^2+y^2+z^2=1$.

3. (a) Determine the values of λ *and* μ such that system of equations.

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu \text{ has}$$

(i) no solution

(ii) unique solution

(iii) infinite no. of solutions.

(b) Find the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(a) Prove that- $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$

(b) Evaluate by changing the order of integration

$$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$$

(a) Prove that-

$div(grad\, r^n) = n(n+1)r^{n-2}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

(b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken round the rectangle bounded by lines

$x = \pm a, y = 0, y = b.$